

# A New Interpretation of MOND Based on Mach Principle and Generalized Equivalence Principle

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**Abstract** A new interpretation is introduced for MOND based on the Sciama's interpretation of Mach principle and an Unruh like effect, in the context of a generalized equivalence principle. It is argued that in a locally accelerated frame with acceleration  $a$  the appearance of a Rindler horizon may give rise to a constant acceleration  $a_0$  as the local properties of cosmological horizon or Hubble length. The total gravitational acceleration inside this frame becomes the combination of  $a$  with  $a_0$ . For  $a \gg a_0$ , the conventional gravitational mass  $m_g$  interacts with the dominant acceleration as  $m_g a$  and application of Sciama's interpretation leads to the standard Newtonian dynamics. For  $a \ll a_0$ , however, a reduced gravitational mass  $\bar{m}_g$  interacts with the dominant acceleration as  $\bar{m}_g a_0$  and the application of Sciama's interpretation on this reduced gravitational mass leads to MOND. This introduces a third proposal for MOND: *The modification of gravitational mass*.

**Keywords** Modified Newtonian dynamics · Sciama's interpretation Mach principle

## 1 Introduction

It is well known that classical Newtonian dynamics fails on galactic scales. There is astronomical and cosmological evidence for a discrepancy between the dynamically measured mass-to-light ratio of any system and the minimum mass-to-light ratios that are compatible with our understanding of stars, of galaxies, of groups and clusters of galaxies, and of superclusters. It turns out that on large scales most astronomical systems have much larger mass-to-light ratios than the central parts. Observations on the rotation curves have turned out that galaxies are not rotating in the same manner as the Solar System. If the orbits of the stars are governed solely by gravitational force, it was expected that stars at the outer edge of

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the disc would have a much lower orbital velocity than those near the middle. In fact, by the Virial theorem the total kinetic energy should be half the total gravitational binding energy of the galaxies. Experimentally, however, the total kinetic energy is found to be much greater than predicted by the Virial theorem. Galactic rotation curves, which illustrate the velocity of rotation versus the distance from the galactic center, cannot be explained by only the visible matter. This suggests that either a large portion of the mass of galaxies was contained in the relatively dark galactic halo or Newtonian dynamics does not apply universally.

The dark matter proposal is mostly referred to Zwicky [1] who gave the first empirical evidence for the existence of the unknown type of matter that takes part in the galactic scale only by its gravitational action. He found that the motion of the galaxies of the clusters induced by the gravitational field of the cluster can only be explained by the assumption of dark matter in addition to the matter of the sum of the observed galaxies. Later, It was demonstrated that dark matter is not only an exotic property of clusters but can also be found in single galaxies to explain their flat rotation curves.

The second proposal results in the modified Newtonian dynamics (MOND), proposed by Milgrom. MOND, as a phenomenological theory, may be interpreted as 1) a modification of inertia, 2) a modification of gravity [2–5]. The well known second law of motion states that an object of mass  $m$  subject to a force  $F$  undergoes an acceleration  $a$  by the simple equation  $F = ma$ . However, it has never been verified for extremely small accelerations which are happening at the scale of galaxies. Based on the modification of inertia the proposition made by Milgrom was the following

$$F = m\mu\left(\frac{a}{a_0}\right)a, \quad (1)$$

$$\mu(x) = \begin{cases} 1 & \text{if } x \gg 1 \\ x & \text{if } x \ll 1, \end{cases} \quad (2)$$

where  $a_0 = 1.2 \times 10^{-10} \text{ m s}^{-2}$  is a proposed new constant. The acceleration  $a$  is usually much greater than  $a_0$  for all physical effects in everyday life, therefore  $\mu(a/a_0) = 1$  and  $F = ma$  as usual. However, at the galactic scale where  $a \ll a_0$  we have the modified dynamics  $F = m\left(\frac{a^2}{a_0}\right)$  leading to a constant velocity of stars on a circular orbit far from the center of galaxies.

There are possible approaches to MOND inertia: deriving effective inertia from interactions with the medium such as vacuum, seeking a new symmetry like Lorentz symmetry that forces a form of the free actions compatible with MOND, and using Mach principle as a new connection between the universe at large and local inertia [2–5]. However, to the authors knowledge, no attention has been paid to study the MOND in the framework of Sciama's interpretation of Mach principle in that inertial forces are interpreted as interaction forces containing an acceleration-dependent term [6–8]. Based on this interpretation, one may find the inertia of an object as the gravitational interaction of its gravitational mass with the distant matter distribution in the universe. So, if according to Milgrom's idea the inertia of an object is to change at very small acceleration, it is reasonable to interpret this modified inertia based on the same Sciama's interpretation.

The main observation in the interpretation of MOND is the connection of  $a_0$  with the cosmic features hinting that MOND might result only in the context of a non-Minkowskian universe, with  $a_0$  reflecting the departure from flatness of space time [2–5].

In this paper, we do not favor MOND to dark matter models and vice versa or do not claim that Sciama's ansatz is correct. However, if MOND has any relation to reality and provided that Sciama's ansatz is reasonable enough to describe the laws of motion, we would like to revisit the MOND from Sciama's interpretation of Mach principle.

## 2 Sciama’s Interpretation of Mach Principle

In this section, we partly use of [9] to establish the Sciama’s interpretation of Mach principle. According to Mach, in any two-body interaction the influence of all other matter inside their causal sphere should be taken into account [10]. For instance, in the process of gravitational interaction of close objects one can replace the distant universe by a spherical shell of the effective mass  $M$  and the effective radius  $R$ . This shell may act as a gravitational Faraday cage inside of which a constant gravitational potential exists as

$$\phi = -\frac{GM}{R}. \tag{3}$$

For an inertial particle the universal field  $\vec{E}$  is zero

$$\vec{E} = -\vec{\nabla}\phi = 0. \tag{4}$$

For accelerated particles, however, similar to the induction law in electrodynamics we have [6–8]

$$\vec{E} = -\vec{\nabla}\phi - \frac{\phi}{c^2}\vec{a} = -\frac{\phi}{c^2}\vec{a}, \tag{5}$$

where  $\vec{a}$  is the acceleration vector and  $c$  is the light velocity. Considering a homogeneous and isotropic distribution of matter in the universe with the average mass density  $\rho$  one may write

$$-\frac{\phi}{c^2} = \frac{G\rho V}{Rc^2} = \frac{2\pi G\rho R^2}{c^2} = \frac{2\pi G\rho}{H^2}, \tag{6}$$

where the volume  $V$  of the universe is considered as a sphere with the Hubble radius

$$R = \frac{c}{H}. \tag{7}$$

Introducing the critical mass density

$$\rho_c = \frac{3H^2}{8\pi G}, \tag{8}$$

and using (6) the following relationship is obtained with a reasonable degree of precision

$$\phi \approx -c^2. \tag{9}$$

It is to be noted that this relationship is exact at the Planck time

$$\phi = \frac{GM_p}{L_p} = -c^2, \tag{10}$$

where  $M_p$  and  $L_p$  are the Planck mass and length, respectively. Therefore, we assume that the condition (9) would be valid in its exact form  $\phi = -c^2$  and hence the gravitational potential would remain unchanged for all the history of the expanding universe [11].

The exertion of the gravitational field of the whole universe (5) on an accelerated particle with the gravitational mass  $m_g$  leads to the standard expression of the inertial force

$$\vec{F} = m_g \vec{E} = m_i \vec{a}, \tag{11}$$

where the concept of inertial mass  $m_i$ , as Mach would desire, is appeared as the measure of the gravitational interaction of the particle with the whole universe as<sup>1</sup>

$$m_i = -m_g \frac{\phi}{c^2}, \quad (12)$$

which using of  $\phi = -c^2$  implies for the equality  $m_g = m_i$  as the strong equivalence principle.

### 3 A Universal Constant Acceleration

The existence of a universal acceleration  $\vec{a}_0$  in the Milgrom's model of dynamics may reveal interesting relation between MOND and Mach Principle in that MOND can represent a feature of the universe as a whole on local dynamics. It is well known that  $\vec{a}_0$  may be expressed in terms of some cosmological quantities. There are, in fact, some quantities with the dimensions of an acceleration, that can be constructed from cosmological parameters as [2–5]

$$a_{ex} \equiv cH_0, \quad (13)$$

$$a_{cu} \equiv \frac{c^2}{R_c}, \quad (14)$$

or

$$a_\lambda \equiv c^2 |\lambda|^{\frac{1}{2}}, \quad (15)$$

where  $H_0$  is the present expansion rate of the universe (the Hubble constant),  $R_c$  is the radius of spatial curvature of the universe and  $\lambda$  is the cosmological constant. The problem of identification of  $\vec{a}_0$  with one of the above candidates would be an important step toward constructing an underlying theory for MOND. The important point is that if  $\vec{a}_0$  as a cosmological quantity do physically exist, then it should become relevant for the particle dynamics once the particle is coupled with the cosmos, as a whole.

From equivalence principle of general relativity we know there is a deep relation between acceleration and gravitation. A frame linearly accelerated relative to an inertial frame in the Minkowskian background of *special relativity* is locally identical to a frame at rest in a gravitational field. It is then obvious that in order to accommodate a universal constant acceleration  $\vec{a}_0$  within this equivalence principle, one may generalize this statement in such a way that large enough above this critical acceleration we get the Newtonian dynamics and small enough below  $\vec{a}_0$  we obtain MOND.

We first suppose a frame which is linearly accelerated by an external agent relative to an inertial frame with acceleration  $\vec{a}$ . The equivalence principle states that as long as the gravitational field in this frame is produced by the local acceleration of this frame with respect to an external inertial frame, the inertial and gravitational masses are indistinguishable. This is known as the strong equivalence principle (SEP). One may wish to show this by resorting to Sciama's interpretation of Mach principle: The observer inside the accelerated frame will

<sup>1</sup>This relation written as  $m_i c^2 + m_g \phi$  implies that the total energy (rest and gravitational) of a particle in the universe is zero, a fact which is certified by theory of inflation in that the total energy (Hamiltonian) of the universe is zero.

observe a massive object freely falling with acceleration  $\vec{a}$ . If, according to Sciama's interpretation of Mach principle, this observer believes in inertia as a kind of interaction with the universe, he should write down this equality according to (5), (11) and using  $\phi = -c^2$  as

$$m_g \vec{E} = m_g \vec{a} = m_i \vec{a}, \quad (16)$$

where  $m_i = m_g$ , as the representation of (SEP), results due to the interpretation of inertial force  $m_i \vec{a}$  as a kind of gravitational interaction  $m_g \vec{a}$ .

Now, suppose there is a constant universal curvature in the neighborhood of every small region in the space. So, if there is a rest frame in this neighborhood, then the isotropy and homogeneity of the universe implies that this constant curvature does not produce a gravitational field with a local preferred direction inside this rest frame. If however, this frame is accelerated by an external agent in a given direction, according to equivalence principle there will be a gravitational field inside the frame with the same magnitude but opposite direction as  $\vec{a}$ . Moreover, the isotropy inside this accelerated frame is broken down due to the preferred gravitational direction and it is plausible that this symmetry breaking may stimulate an extra local gravitational field originated by the constant universal curvature, with the same direction as that of  $\vec{a}$ . Note that the isotropy is broken down just for accelerated frames and not inertial ones with uniform motions. This may be similar to the well known Unruh effect in that a thermal effect appears merely in an accelerated frame which breaks down the isotropy of the vacuum due to the appearance of an apparent event horizon. The Rindler coordinate system or frame describes a uniformly accelerated frame of reference in Minkowski space. It is well known that the Rindler spacetime has a horizon and gives the local properties of *black holes* and *cosmological horizons*. We know the Unruh effect is the near-horizon form of the Hawking radiation. In the same way, one may suppose a non trivial gravitational effect appears in the accelerated frame as a local properties of *cosmological horizon* or Hubble length  $H_0$  which is related to the constant curvature of the universe through the  $R_c$ , namely the radius of spatial curvature of the universe.

One may also consider the emergence of the nonzero universal acceleration  $a_0$  as an Unruh like effect, within the framework of holographic principle. The holographic principle is a property of quantum gravity which states that the description of a volume of space can be considered as encoded on a boundary, like a gravitational horizon, to that region. This theory suggests that the entire universe can be seen as a two-dimensional information structure "painted" on the cosmological horizon, namely the universe may be like a gigantic hologram. In this regard, one may consider the *local* acceleration  $a$  as the motion with respect to the neighborhood volume of space, and the *universal* acceleration  $a_0$  as the motion with respect to the entire boundary of the observable universe, namely the cosmological horizon.

The appearance of this extra gravitational field which is related to  $H_0$  or  $R_c$  may account for a constant universal acceleration  $a_0 \approx a_{ex}$  or  $a_0 \approx a_{cu}$  inside the accelerated frame, as Milgrom has proposed [2–5]. Therefore, the observer in this frame will observe an interaction of the gravitational mass of the object with the total acceleration

$$\vec{a}_T = \vec{a} + \vec{a}_0. \quad (17)$$

This gravitational interaction must be balanced by the inertial force  $m_i \vec{a}$  where  $m_i$  is the *constant* inertial mass which is coupled just to  $\vec{a}$  with inertial origin (produced by an external agent as mentioned above) and has nothing to do with  $\vec{a}_0$  which has a gravitational origin as a local properties of cosmological horizon (produced by an Unruh like effect).

In fact,  $\vec{a}_0$  makes an absolute demarkation between the gravitational and inertial masses and they are no longer equivalent inside this locally accelerated frame. Since this difference is caused by an extra gravitational field  $\vec{a}_0$  and the inertial properties of an object do not change (the external accelerating agent does no change) it is plausible to assume that in this frame the inertial mass of the object is perfectly constant and it is the behavior of gravitational mass which is changed due to the appearance of  $\vec{a}_0$ .

For  $a \gg a_0$  the dominant acceleration is  $\vec{a}_T = \vec{a}$ . So, the gravitational interaction occurs between the gravitational mass  $m_g$  and  $\vec{a}$  which is to be balanced by the inertial force as

$$m_g \vec{a} = m_i \vec{a}. \tag{18}$$

This case was already discussed in the previous section and led to the Newtonian dynamics. For  $a \ll a_0$  the dominant acceleration is  $\vec{a}_T = \vec{a}_0$ . In this case, the gravitational interaction occurs between an effective gravitational mass  $\vec{m}_g$  and  $\vec{a}_0$  in order to be balanced with the same (unchanged) inertial force as

$$\vec{m}_g \vec{a}_0 = m_i \vec{a}, \tag{19}$$

which leads to a *reduced* gravitational mass

$$\vec{m}_g = m_i \frac{a}{a_0}. \tag{20}$$

The important and key point is that the reduced gravitational mass  $\vec{m}_g$  is interpreted as the *gravitational charge* against the universal gravitational field  $\vec{a}_0$  and all other gravitational sources with cosmological origin. On the other hand, the conventional gravitational mass  $m_g$  is interpreted as the *gravitational charge* against the local gravitational fields  $\vec{a}$ . Now, suppose our observer in the accelerated frame would like to use again Sciama’s interpretation of Mach principle to evaluate the gravitational interaction of the object with the cosmological potential  $\phi$ . To this end, using (18) and  $\phi = -c^2$  he should write down as follows

$$\vec{m}_g \vec{E} = -m_i \frac{a}{a_0} \frac{\phi}{c^2} \vec{a} = m_i \frac{a}{a_0} \vec{a}, \tag{21}$$

where, as mentioned above, the reduced gravitational mass  $\vec{m}_g$  as the *gravitational charge* against the gravitational sources with cosmological origin interacts with the gravitational field  $\vec{E}$  which has a cosmological origin  $\phi$ . The resultant gravitational interaction manifests as the law of motion in the modified dynamics which Milgrom has proposed. The observer in the accelerated frame may apply this inertial force for the freely falling object which appears to be under a local real central gravitational force  $F \sim r^{-2}$  as

$$\frac{Gm_g M}{r^2} = m_i \frac{a^2}{a_0}. \tag{22}$$

Note that in the L.H.S of force law of gravitation, the conventional gravitational mass  $m_g$  is appeared since according to our assumption it is interacting with the *local* gravitational force  $F \sim r^{-2}$ . Therefore, using (SEP) as  $m_g = m_i$  the observer arrives at

$$a = \frac{\sqrt{GMa_0}}{r}. \tag{23}$$

According to equivalence principle the results obtained in this accelerated frame is the same for an observer at rest in a gravitational field. Therefore, the central (freely falling) acceleration of an object for which  $a \ll a_0$  at a distance  $r$  from the center of a galaxy is the same as (23) which may justify the galaxy's rotation curve.

#### 4 Conclusion

In this paper, we have interpreted the MOND based on the Sciama's interpretation of Mach principle, an Unruh like effect and the equivalence principle. We have shown that the equivalence principle may be generalized to incorporate the constant acceleration  $a_0$  which is produced in an Unruh like effect. The price is paid by introducing a reduced gravitational mass. At large accelerations  $a \gg a_0$ ,  $a$  being the acceleration of the local frame, the dominant interaction occurs between the *conventional* gravitational mass and  $a$  so that application of Sciama's ansatz leads to Newtonian dynamics. At small accelerations  $a \ll a_0$ , the dominant interaction occurs between the *reduced* gravitational mass and  $a_0$  so that application of Sciama's ansatz leads to Modified Newtonian dynamics. This approach introduces a third proposal for MOND as *modification of gravitational mass* beside the two previous proposals, namely *modification of inertia* and *modification of gravity*.

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